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A METHOD OF ESTIMATING THE PARAMETERS OF TUNED MASS DAMPERS FOR SEISMIC APPLICATIONS

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SUMMARY

The optimum parameters of tuned mass dampers (TMD) that result in considerable reduction in the response of structures to seismic loading are presented. The criterion used to obtain the optimum parameters is to select, for a given mass ratio, the frequency (tuning) and damping ratios that would result in equal and large modal damping in the first two modes of vibration. The parameters are used to compute the response of several single and multi-degree-of-freedom structures with TMDs to different earthquake excitations. The results indicate that the use of the proposed parameters reduces the displacement and acceleration responses significantly. The method can also be used in vibration control of tall buildings using the so-called 'mega-substructure configuration', where substructures serve as vibration absorbers for the main structure. It is shown that by selecting the optimum TMD parameters as proposed in this paper, significant reduction in the response of tall buildings can be achieved. © 1997 by John Wiley & Sons, Ltd. Earthquake eng. struct. dyn. 26: 617–635, 1997.

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INTRODUCTION

The tuned mass damper (TMD) is a passive energy absorbing device consisting of a mass, a spring, and a viscous damper attached to a vibrating system to reduce undesirable vibrations. According to Ormondroyd and Den Hartog,[1] the use of TMDs was first suggested in 1909. Since then, much research has been carried out to investigate their effectiveness for different dynamic loading applications. Tuned mass dampers are effective in reducing the response of structures to harmonic[2] or wind[3,4] excitations. TMDs have been installed in high rise buildings to reduce wind-induced vibrations. Examples include: the 244 m high John Hancock Tower in Boston[5] with a TMD consisting of two $2{\cdot}7 \times 10^5$ kg (300 t) lead and steel blocks; the 280 m high Citicorp Center Office Building in New York City[6] with a TMD using a $3{\cdot}6 \times 10^5$ kg (400 t) concrete block, and the Terrace on the Park Building in New York City,[7] where a TMD was installed to reduce the vibrations induced by dancing. For seismic applications, however, there has not been a general agreement on the efficiency of TMD systems to reduce the structural response.

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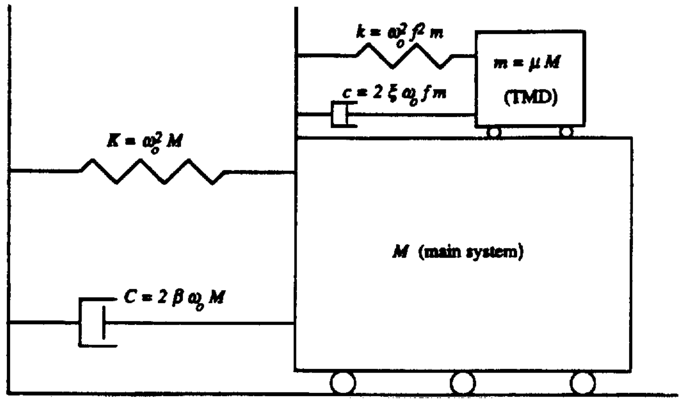


Figure 1. Tuned mass damper mounted on a main structure

This paper briefly reviews studies on the use of TMDs for seismic applications and proposes a method for selecting the TMD parameters by providing equal and large damping ratios in the complex modes† of vibration. The optimum parameters are formulated in terms of the mass ratio of the TMD, the damping ratio and mode shapes of the structure. To show the effectiveness of the proposed method, the response of several single and multi-degree-of-freedom structures, with and without TMDs, to different ground excitations are presented and compared to those from other methods. The method is also used to compute the tuning and damping ratios of substructures utilized as vibration absorbers in tall buildings. This concept, referred to as 'mega-substructure configuration' by Feng and Mita[8] uses no external devices nor additional masses to control vibrations. Comparisons with responses using their method are presented to demonstrate the effectiveness of the method proposed herein.

SUMMARY OF PREVIOUS WORK

A typical tuned mass damper consists of a mass m which moves relative to the structure and is attached to it by a spring (with stiffness k) and a viscous damper (with coefficient c) as shown in Figure 1. A tuned mass damper is characterized by its tuning, mass, and damping ratios. The tuning ratio f is defined as the ratio of the fundamental frequency of the TMD ω_t to that of the structure ω_0. Thus,

$$f = \omega_t / \omega_0 \tag{1}$$

The mass ratio μ is

$$\mu = m / M \tag{2}$$

where M is the total mass of a SDOF structure or the generalized mass for a given mode of vibration of a MDOF structure computed for a unit modal participation factor. The damping ratio of the TMD is given by

$$\xi = c / 2m\omega_t \tag{3}$$

Several investigators have studied the effect of optimum TMD parameters f and ξ for a given μ on reducing the response of structures to earthquake loading. There has not been a general agreement, however, on the effectiveness of TMDs in reducing structural response to seismic loading. The following is a brief summary of the studies.

† Because of non-proportional damping, the analysis of TMD systems lends itself to complex modal analysis

Gupta and Chandrasekaren[9] studied the influence of several TMDs with elastic–plastic properties on the response of SDOF structures subjected to the S21W component of the Taft accelerogram, Kern County earthquake, 1952. Their study showed that TMDs are not as effective in reducing the response of structures to earthquake excitations as they are for sinusoidal loads. Kaynia *et al.*[10] used an ensemble of 48 earthquake accelerograms to investigate the effect of TMDs on the fundamental mode response. They found that the optimum reduction in response is achieved at $f = 1$ and that increasing the period and damping of the structure decreases the effectiveness of TMDs. They concluded, however, that in general TMDs are less effective in reducing the seismic response of structures than previously thought. Sladek and Klingner[11] used the Den Hartog[2] method to select the parameters f and ξ for a TMD placed on the top floor of a 25-storey building. The basis for the Den Hartog method is to minimize the response to sinusoidal loading which for an undamped system results in the following TMD parameters:

$$f = \frac{1}{1+\mu} \quad \text{and} \quad \xi = \sqrt{\frac{3\mu}{8(1+\mu)}} \tag{4}$$

The analysis of the 25-storey building subjected to the S00E component of the El Centro accelerogram, the Imperial Valley earthquake, 1940 revealed that the TMD was not effective in reducing the response of the building.

The first successful analysis of TMD for seismic loading was introduced by Wirsching and Yao[12] where they computed the first-mode response to a non-stationary ground acceleration for a five- and a ten-storey building with 2 per cent damping. They selected a TMD mass equal to one-half the mass of a typical floor and a tuning ratio $f = 1$. Considerable reduction in response was obtained with a TMD damping ratio $\xi = 0{\cdot}20$. Later, Wirsching and Campbell[13] used an optimization method to calculate the TMD parameters for 1-, 5- and 10-storey buildings subjected to a stationary white noise base acceleration. They observed that TMDs were quite effective in reducing the response.

Dong[14] observed that the light wing of a building can act as a vibration absorber for the main building and reduce its seismic response significantly, while the wing itself may experience large displacements. Ohno *et al.*[15] presented a method for tuning TMDs so that the mean square acceleration response of the main structure is minimized. They assumed that the acceleration power spectral density of the earthquake ground motion at the base is constant for a certain frequency range. Jagadish *et al.*[16] analysed a two-storey structure with a bilinear material behaviour subjected to the S69E component of the Taft accelerogram, Kern County earthquake, 1952 with the top floor functioning as a vibration absorber for the lower one. They observed that for $f = 0{\cdot}8$–$1{\cdot}0$, a reduction of 50 per cent in the ductility demand for the lower storey can be achieved. They also introduced the concept of 'expendable top storey' where the top floor can absorb a major portion of the seismic energy and experience damage, thereby, reducing the response of the lower stories. Such a concept juxtaposes the 'soft first storey' concept where the earthquake energy is absorbed at the base or the first level. The soft first storey approach, however, is not practical and may jeopardize the stability and safety of the structure.

Numerous studies on the applicability of TMDs for seismic applications were carried out by Villaverde,[17,18] Villaverde and Koyama,[19] and Villaverde and Martin[20] where it was found that TMDs performed best when the first two complex modes of vibration of the combined structure and damper have approximately the same damping ratios as the average of the damping ratios of the structure and the TMD. To achieve this, Villaverde[17] found that the TMD should be in resonance with the main structure ($f = 1$) and its damping ratio be

$$\xi = \beta + \Phi\sqrt{\mu} \tag{5}$$

where β is the damping ratio of the structure, μ is the ratio of the mass of the absorber to the generalized mass of the structure in a given mode of vibration (usually the fundamental mode) and Φ is the amplitude of the mode shape at the TMD location. Both μ and Φ are computed for a unit modal participation factor. This method was used in several 2D and 3D analyses of buildings and cable-stayed bridges under different ground excitations and was found effective, numerically and experimentally, in reducing the response. It will be discussed later that Villaverde's formulation does not result in equal dampings in the first two modes of vibration, especially for large mass ratios. More recently, Miyama[21] argued that TMDs with a small mass (less than 2 per cent of the first mode generalized mass) are not effective in reducing the response of buildings to earthquake excitations. He suggested that most of the seismic energy should be absorbed by the top storey so that the other stories would remain undamaged. The top storey should have appropriate strength, ductility and supplemental damping to resist the loads. Numerical results indicate that it is possible to obtain 80 per cent energy absorption with a mass ratio of 5 per cent by tuning the frequency of the top storey to that of the structure.

From the above discussions, it seems that TMDs can be effective in reducing the response of structures to seismic loads. The problem is how to find the optimum TMD parameters in order to achieve the greatest reduction in response. In the following sections, an improvement to the method introduced by Villaverde is presented and new equations are formulated to ensure that the first two modes of vibration of the structure with TMD will have equal damping ratios which are greater than $(\xi + \beta)/2$. Numerical results are presented to illustrate the effectiveness of the improved method in determining the TMD parameters for seismic applications.

TMD FOR SDOF STRUCTURES

For a SDOF structure with a TMD (Figure 1), the system matrix A in terms of the natural frequency and damping ratio (ω_0 and β) of the structure, and the mass, tuning, and damping ratios (μ, f, and ξ) of the TMD is

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\omega_0^2 f^2 & \omega_0^2 f^2 & -2\omega_0 f\xi & 2\omega_0 f\xi \\ \omega_0^2 \mu f^2 & -\omega_0^2(1+\mu f^2) & 2\omega_0 \mu f\xi & -2\omega_0(\mu f\xi + \beta) \end{bmatrix} \tag{6}$$

The eigenvalue problem $|A\text{-}\lambda I|$ results in the following fourth-order equation:

$$\left(\frac{\lambda}{\omega_0}\right)^4 + [2f\xi(1+\mu) + 2\beta]\left(\frac{\lambda}{\omega_0}\right)^3 + [1 + \mu f^2 + f^2 + 4f\xi\beta]\left(\frac{\lambda}{\omega_0}\right)^2 + 2f(\xi + f\beta)\left(\frac{\lambda}{\omega_0}\right) + f^2 = 0 \tag{7}$$

The solution of equation (7) is in complex conjugate pairs with the following complex eigenvalues:

$$\lambda_{r,r+1} = -\omega_r \xi_r \pm \mathrm{i}\omega_r\sqrt{1-\xi_r^2}\ , \quad r = 1,3 \tag{8}$$

where λ_r is the rth eigenvalue, ω_r and ξ_r are the natural frequency and damping ratio of the system in the rth mode, and i is the unit imaginary number ($\mathrm{i} = \sqrt{-1}$). Villaverde[17] showed that for a TMD to be effective, the damping ratios in the two complex modes of vibration, ξ_1 and ξ_3 should be approximately equal to the average of the damping ratios of the structure and the TMD, i.e. $\xi_1 \cong \xi_3 \cong (\xi + \beta)/2$. To achieve this criterion, it was shown analytically[17,19] that the TMD should be in resonance with the main system ($f = 1$) and its damping ratio should satisfy equation (5). Numerical results, however, show that such formulation is valid only for mass ratios smaller than approximately 0·005. For mass ratios larger than 0·005, significant difference in the two modal dampings exists for a typical structure with a damping ratio $\beta = 0{\cdot}05$ (see Table I). Consequently, another procedure to achieve equal damping in the two vibration modes is proposed in this paper.

Table I. Complex mode damping ratios computed by the Villaverde method for a structure with damping $\beta = 0{\cdot}05$

μ	ξ	$(\xi + \beta)/2$	ξ_1	ξ_3
0·005	0·1207	0·0854	0·0983	0·0727
0·010	0·1500	0·1000	0·1207	0·0801
0·020	0·1914	0·1207	0·1544	0·0888
0·050	0·2736	0·1618	0·2281	0·1019
0·100	0·3662	0·2081	0·3218	0·1111

The proposed procedure searches numerically for the optimum values of f and ξ (the optimum values are those which result in approximately equal damping ratios $\xi_1 \cong \xi_3$) corresponding to a desired mass ratio μ. Since the eigenvalues in equation (7) are normalized to ω_0, the optimum parameters f and ξ are independent of the natural frequency (ω_0) of the main system. To determine the optimum values of f and ξ for a given μ and β, the complex eigenvalue problem $|A\text{-}\lambda I|$ is solved in the following manner: for a given damping ratio β and

Table II. Optimum TMD tuning and damping ratios for three structural damping ratios

Mass ratio	$\beta = 0$		$\beta = 0{\cdot}02$		$\beta = 0{\cdot}05$	
μ	f	ξ	f	ξ	f	ξ
0·000	1·0000	0·0000	1·0000	0·0200	1·0000	0.0500
0·005	0·9950	0·0705	0·9936	0·0904	0·9915	0·1199
0·010	0·9901	0·0995	0·9881	0·1193	0·9852	0·1488
0·015	0·9852	0·1216	0·9828	0·1412	0·9792	0·1707
0·020	0·9804	0·1400	0·9776	0·1596	0·9735	0·1889
0.025	0.9756	0.1562	0.9726	0.1757	0.9680	0.2048
0.030	0.9709	0·1707	0·9676	0·1900	0·9626	0·2190
0·035	0·9662	0·1839	0·9626	0·2032	0·9573	0·2320
0·040	0·9615	0·1961	0·9578	0·2153	0·9521	0·2440
0·045	0·9569	0·2075	0·9530	0·2266	0·9470	0·2551
0·050	0·9524	0·2182	0·9482	0·2372	0·9420	0·2656
0·055	0·9479	0·2283	0·9435	0·2472	0·9370	0·2754
0·060	0·9434	0·2379	0·9389	0·2567	0·9322	0·2848
0·065	0·9390	0·2470	0·9343	0·2658	0·9274	0·2937
0·070	0·9346	0·2558	0·9298	0·2744	0·9226	0·3022
0·075	0·9302	0·2641	0·9253	0·2827	0·9179	0·3103
0·080	0·9259	0·2722	0·9209	0·2906	0·9133	0·3181
0·085	0·9216	0·2799	0·9165	0·2983	0·9087	0·3257
0·090	0·9174	0·2873	0·9122	0·3056	0·9042	0·3329
0·095	0·9132	0·2945	0·9079	0·3128	0·8998	0·3399
0·100	0·9091	0·3015	0·9036	0·3196	0·8954	0·3466
0·105	0·9050	0·3083	0·8994	0·3263	0·8910	0·3532
0·110	0·9009	0·3148	0·8952	0·3328	0·8867	0·3595
0·115	0·8969	0·3212	0·8911	0·3390	0·8824	0·3656
0·120	0·8929	0·3273	0·8870	0·3451	0·8782	0·3716
0·125	0·8889	0·3333	0·8830	0·3511	0·8741	0·3774
0·130	0·8850	0·3392	0·8790	0·3568	0·8699	0·3831
0·135	0·8811	0·3449	0·8750	0·3624	0·8658	0·3886
0·140	0·8772	0·3504	0·8710	0·3679	0·8618	0·3939
0·145	0·8734	0·3559	0·8671	0·3733	0·8578	0·3991
0·150	0·8696	0·3612	0·8633	0·3785	0·8538	0·4042

Note: $\xi_1 \cong \xi_3$ and $\omega_1 \cong \omega_3$

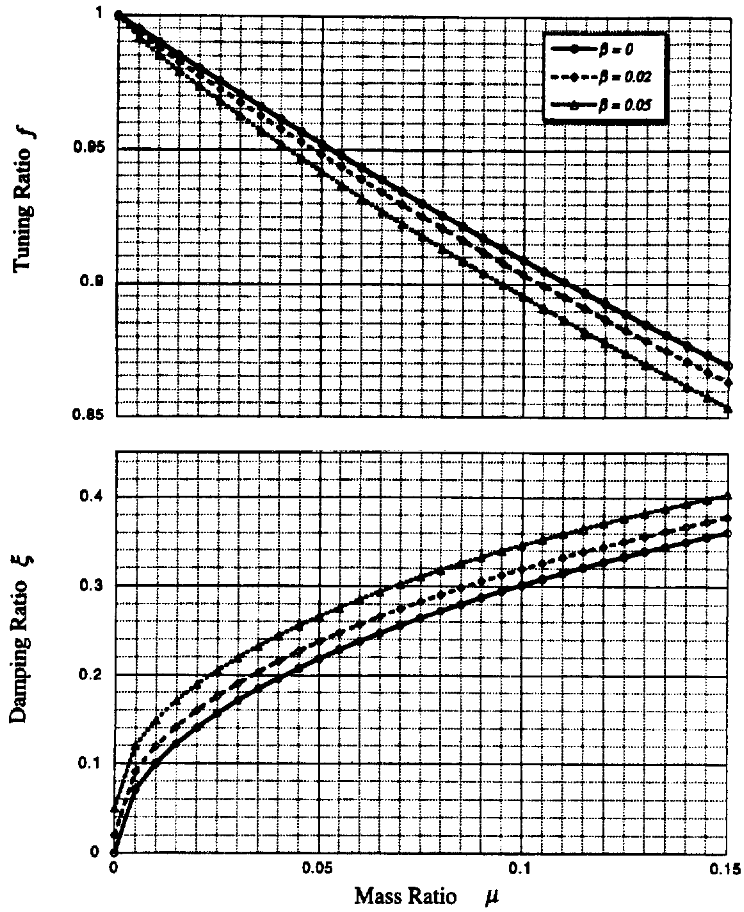


Figure 2. Optimum TMD parameters for different mass and damping ratios

for each mass ratio μ, the values of f and ξ are varied, matrix A is formed, and its eigenvalues are computed. The optimum values are determined when the difference between the two damping ratios ξ_1 and ξ_3 is the smallest. The procedure was used for systems with damping ratios $\beta = 0$·, 0·02, and 0·05 and mass ratios μ between 0·005 and 0·15 with increments of 0·005. It was found that the optimum TMD parameters result in approximately equal modal damping ratios ($\xi_1 \cong \xi_3$) greater than $(\xi + \beta)/2$ and equal modal frequencies ($\omega_1 \cong \omega_3$). The optimum ratios are presented in Table II. Figure 2 shows the optimum parameters f and ξ for different mass ratios and the three structural damping ratios β. The figure indicates that the higher the damping ratio of the structure, the lower is the tuning ratio and the higher is the TMD damping ratio. The figure may be used to select the TMD parameters by estimating its mass, computing the mass ratio μ, and then determining the tuning and damping ratios f and ξ. Figure 3 shows the modal frequencies and dampings for the structure with TMD. It is observed from the figure that the higher the mass ratio, the higher the damping in the modes. From Table II and Figures 2 and 3, it is evident that increasing the mass ratio μ requires a decrease in the tuning ratio f and an increase in the damping ratio ξ, thus resulting in a higher damping in the two modes of vibration.

For design purposes, it may be convenient to present the optimum TMD parameters by simple equations rather than tables. Curve fitting was used to find f and ξ in terms of μ and β. For an undamped structure, the

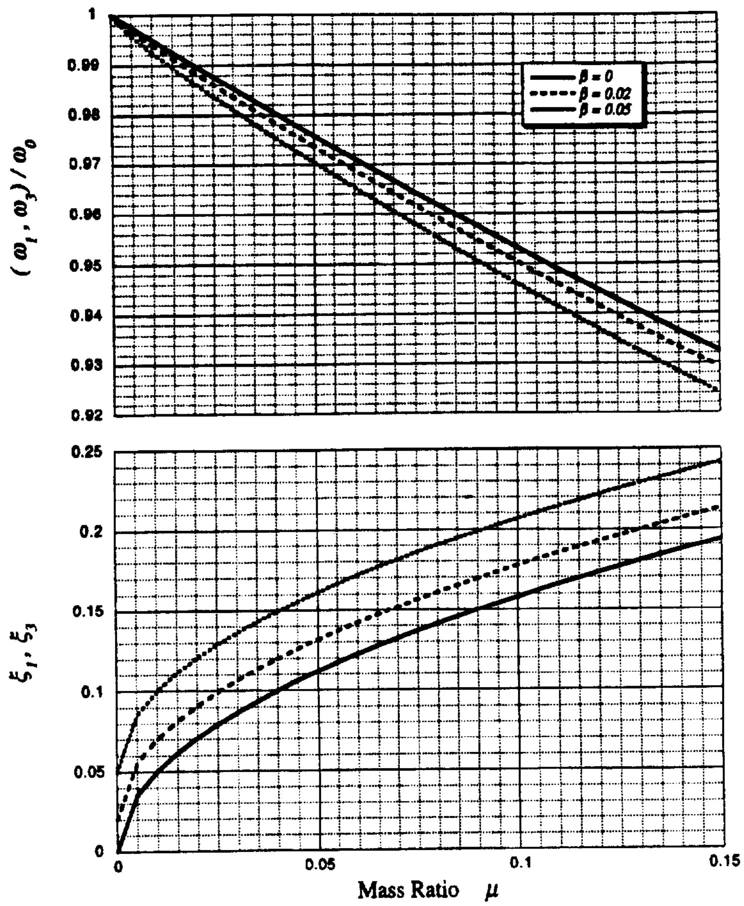


Figure 3. Natural frequencies and damping ratios in the first two modes

tuning ratio f is found to be equal to $1/(1 + \mu)$ and the damping ratio ξ equal to $\sqrt{\mu/(1 + \mu)}$. For a damped structure, the following equations give close approximations to the f and ξ values presented in Table II:

$$f = \frac{1}{1 + \mu}\left[1 - \beta\sqrt{\frac{\mu}{1 + \mu}}\right] \tag{9}$$

and

$$\xi = \frac{\beta}{1 + \mu} + \sqrt{\frac{\mu}{1 + \mu}} \tag{10}$$

These equations result in a maximum error of approximately 0·2 per cent in f and 0·4 per cent in ξ.

Numerical studies

To examine the effectiveness of the proposed procedure in determining the TMD parameters for seismic excitations, 30 SDOF structures with periods between 0·1 and 3·0 s with increments of 0·1 s were analysed with and without TMDs. The structures had damping ratios $\beta = 0{\cdot}02$ and 0·05 and the mass ratios were selected to vary between 0·02 and 0·10 with increments of 0·02. The TMD parameters used were those presented in Table II. The structures were subjected to a set of 52 horizontal components of accelerograms from 26 stations in the western United States (Appendix I). These records include a wide range of earthquake magnitudes (5·2–7·7), epicentral distances (6–127 km), peak ground accelerations (0·044g to 1·172g), and two soil conditions (rock and alluvium). The response (displacement or acceleration) ratio is computed as the ratio of the peak response of the structure with TMD to the peak response without TMD. The stroke ratio is defined as the peak stroke length (displacement of TMD relative to that of the structure) divided by the peak displacement of the structure. The mean displacement and acceleration response ratios, and the mean stroke ratio for the 30 structures, the five mass ratios, and the 52 records are shown in Figure 4 for a damping ratio of 0·02 and in Figure 5 for a damping ratio of 0·05.

The following observations can be made from Figures 3–5:

(a) Reductions in displacement and acceleration responses can be achieved with a TMD, especially for structures with low damping ratios $\beta = 0{\cdot}02$ (Figures 4 and 5).

(b) Increasing the mass ratio decreases the response (Figures 4 and 5). This is expected since increasing the mass ratio results in a higher TMD damping ratio, and consequently, a higher damping in the two modes of vibration.

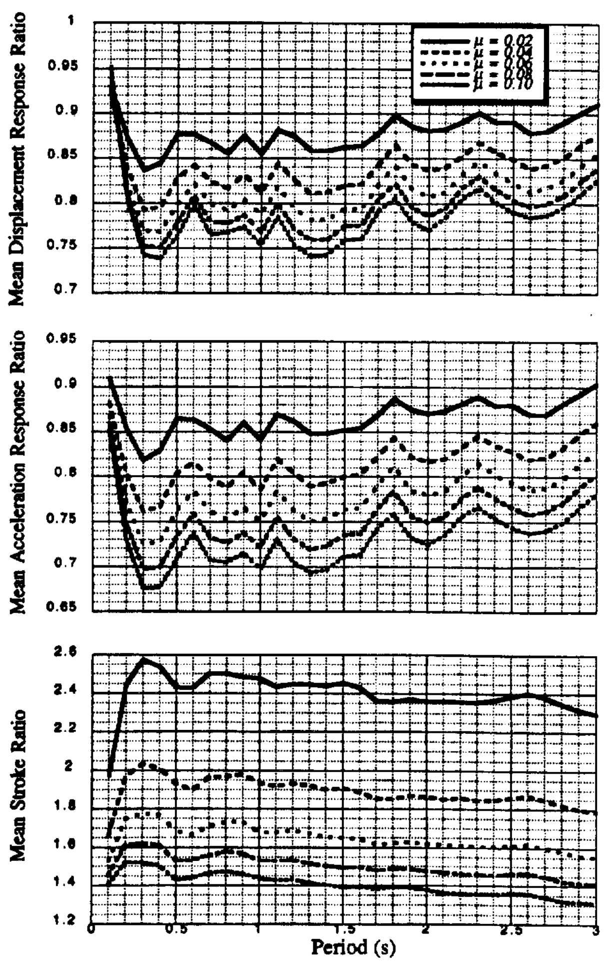


Figure 4. Mean response of SDOF structures with TMDs with 0.02 damping

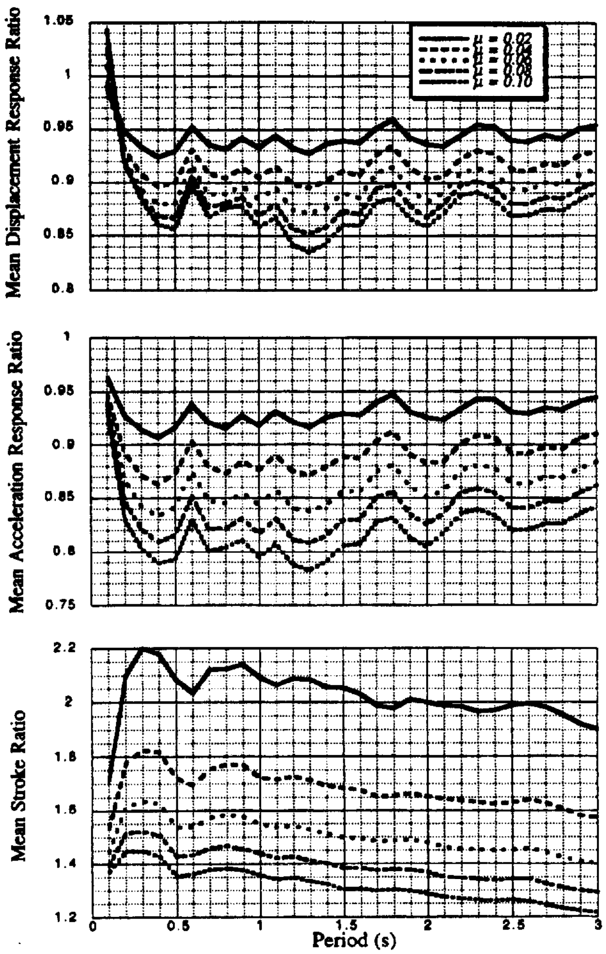


Figure 5. Mean response of SDOF structures with TMDs with 0.05 damping

(c) For a mass ratio of 0·04 (Figure 3), the structure with a $\beta = 0{\cdot}02$ will have a damping ratio of 0·12 in the first two modes (six times greater than the damping of the structure), whereas for a $\beta = 0{\cdot}05$, the first two modes will have a damping ratio of 0·15 (only three times the damping of the structure). Therefore, TMDs are more effective for structures with small damping ratios.
(d) For rigid structures, i.e. structures with periods 0·1–0·2 s (Figures 4 and 5), TMDs are not effective. The use of a higher mass ratio is not desirable and may be even detrimental to the structure, especially for a damping ratio of 0·05.
(e) As expected, for systems with small damping ratios, the stroke length is larger. This must be accounted for in design (Figures 4 and 5).

To examine the dispersion of the results obtained from the 52 records, the coefficient of variation (COV) (standard deviation divided by the mean) was computed for various cases. Figure 6 presents the COV for the displacement response ratio for structures with a damping of 0·05. The figure shows that the larger the mass ratio, the larger is the dispersion of the results. The COV, however, for all periods and mass ratios is less than 0·16. Similar values were obtained for mean acceleration response and stroke lengths.

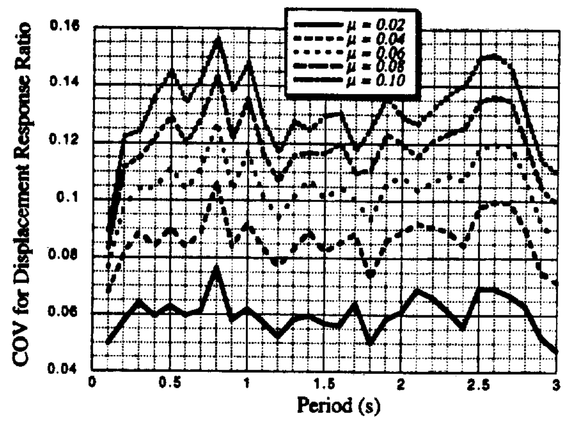


Figure 6. Coefficient of variation for displacement ratio for SDOF structures with 0.05 damping

Comparison with Villaverde's method

The method proposed herein is compared with that introduced by Villaverde. The comparison is carried out for two SDOF structures with different periods T, dampings β and mass ratios μ as shown in Table III. Four accelerograms — the S00E component of El Centro from the Imperial Valley earthquake, 1940; the S90W component of El Centro from the Borrego Mountain earthquake, 1968; the N00W component at 8244 Orion Boulevard and the N00E component at 7080 Hollywood Boulevard from the San Fernando Earthquake, 1971— were used in the analysis. Table III indicates that the method proposed in this paper results in a lower response than Villaverde's method. It should be noted that for a given mass ratio, the proposed method results in a lower stiffness (compare $f = 1$ with $f < 1$ in equation (9)) and a lower damping coefficient (compare ξ in equation (5) with ξ in equation (10)) than those used by Villaverde, and yet, gives smaller displacements and accelerations up to 14 per cent. The reason that the proposed method produces better reduction in the response is that it results in approximately equal damping in the first two modes, whereas, the method by Villaverde results in unequal damping. The difference between the two modal dampings in Villaverde's method is more pronounced when the mass ratios are large. Consequently, the mode with the lower damping increases the overall structural response.

The results in Table III for the structure with a period $T = 0{\cdot}50$ s, a damping ratio $\beta = 0{\cdot}05$, and a mass ratio $\mu = 0{\cdot}12$ subjected to the S00E component of the El Centro accelerogram, 1940 show practically no reduction in the displacement response with a TMD. The reason is that the addition of the TMD corresponds to $\omega_1/\omega_0 = 0{\cdot}93$ (Figure 3) indicating a shift in the period of the structure from 0·50 to 0·54 s and the damping ratio from 0·05 to 0·22 (see Figure 3). An examination of the response spectrum of this record reveals that the shift in the period results in a higher response. It should be noted that TMD parameters selected by the Villaverde's method resulted in a higher displacement response than that without a TMD.

TMD FOR MDOF STRUCTURES

In this section, the optimum TMD parameters for MDOF structures are formulated and the effectiveness of these parameters in reducing the response to earthquake loading is examined. For an n degree of freedom structure with a TMD attached to one of the floors, there are $n + 1$ pairs of complex conjugate modes. For a MDOF structure, the mass ratio is computed as the ratio of the TMD mass to the generalized mass for the

Table III. Response of SDOF systems with and without TMDs using the method by Villaverde[17] and the method proposed in this study

Structural properties	Method of analysis	Imperial valley, 1940 El Centro		Borrego mountain, 1968 El Centro		San Fernando, 1971 Orion Boulevard		San Fernando, 1971 Hollywood Boulevard	
		X_{max} (mm)	a_{max} (g)	X_{max} (mm)	a_{max} (g)	X_{max} (mm)	a_{max} (g)	X_{max} (mm)	a_{max} (g)
$T = 0{\cdot}25$ s	No TMD	18·6	1·200	4·3	0·275	10·1	0·648	3·7	0·239
$\beta = 0{\cdot}02$	Villaverde	13·3	0·768	2·7	0·155	10·3	0·580	3·8	0·216
$\mu = 0{\cdot}10$	This study	12·5	0·748	2·4	0·141	09·4	0·539	3·5	0·202
$T = 0{\cdot}50$ s	No TMD	51·6	0·836	7·9	0·128	34·4	0·556	13·5	0·218
$\beta = 0{\cdot}05$	Villaverde	54·5	0·767	6·5	0·093	38·9	0·537	12·1	0·174
$\mu = 0{\cdot}12$	This study	51·0	0·732	6·0	0·089	33·9	0·498	10·4	0·156

Table IV. Properties of the three MDOF structures used in the analysis

(a) Properties of the ten-storey frame

Stiffness × 10^3 kN/m	Mass × 10^3 kg	First mode shape
34·31	98	1·359
37·43	107	1·321
40·55	116	1·248
43·67	125	1·146
46·79	134	1·019
49.91	143	0·871
53·02	152	0·708
56·14	161	0·534
52·26	170	0·355
62·47	179	0·175

$\beta = 0{\cdot}02$
$\omega_{01} = 0{\cdot}5$ Hz
$M_1 = \phi_1^T [M] \phi_1 = 1109 \times 10^3$ kg

(b) Properties of the six-storey frame

Stiffness × 10^6 kN/m	Mass × 10^6 kg	First mode shape
4·5	8·0	1·327
5·5	8·0	1·186
7·5	8·0	0.966
8·0	8·0	0.743
9·0	8·0	0·489
10·0	8·0	0.238

$\beta = 0{\cdot}05$
$\omega_{01} = 1{\cdot}23$ Hz
$M_1 = \phi_1^T [M] \phi_1 = 39598 \times 10^3$ kg

(c) Properties of the three-storey frame

Stiffness × 10^3 kN/m	Mass × 10^3 kg	Frst mode shape
36·0	100·0	1·231
38·0	100·0	0·965
41·0	100·0	0·515

$\beta = 0$
$\omega_{01} = 1{\cdot}41$ Hz
$M_1 = \phi_1^T [M] \phi_1 = 271 \times 10^3$ kg

Note: (1) Properties of floors are shown from top to bottom
(2) First mode shapes and generalized masses are computed for a unit modal participation factor

fundamental mode for a unit modal participation factor

$$\mu = \frac{m}{\phi_1^T[M]\phi_1} \tag{11}$$

where $[M]$ is the mass matrix and ϕ_1 is the fundamental mode shape normalized to have a unit participation factor. A procedure similar to that for SDOF systems is used to determine the optimum f and ξ that would result in approximately equal frequencies and damping ratios in the first two modes. Numerical studies were carried out using three MDOF structures: a ten-, a six-, and a three-storey building. The structures are assumed to have the following damping ratios in the first mode only: 0·02 for the ten-storey, 0·05 for the six-storey, and zero for the three-storey building. The properties of the three frames together with their fundamental mode dynamic characteristics are given in Table IV. For each frame, a TMD is attached to the top floor to control the response. The mass ratio μ is assumed to be 0·05 for the ten-storey, 0·075 for the six-storey, and 0·10 for the three-storey building. The optimum values of f and ξ for the three structures are given in Table V along with the resulting damping ratios in the first two modes of vibration. As shown in the table, the damping ratios are extremely close to each other and are greater than $(\xi + \beta)/2$. It should be mentioned that the TMDs attached to the structures affected only the damping in the first two modes and had no effect on the other modes which were assumed to have a zero damping.

It was found that the tuning ratio f for a MDOF system is nearly equal to the tuning ratio for a SDOF system for a mass ratio of $\mu\Phi$, where Φ is the amplitude of the first mode of vibration for a unit modal participation factor computed at the location of the TMD, i.e. $f_{MDOF}(\mu) = f_{SDOF}(\mu\Phi)$. The equation for the tuning ratio is obtained from equation (9) by replacing μ by $\mu\Phi$. Thus,

$$f = \frac{1}{1+\mu\Phi}\left[1 - \beta\sqrt{\frac{\mu\Phi}{1+\mu\Phi}}\right] \tag{12}$$

The TMD damping ratio is also found to correspond approximately to the damping ratio computed for a SDOF system multiplied by Φ, i.e. $\xi_{MDOF}(\mu) = \Phi\xi_{SDOF}(\mu)$. The equation for the damping ratio is obtained by multiplying equation (10) by Φ

$$\xi = \Phi\left[\frac{\beta}{1+\mu} + \sqrt{\frac{\mu}{1+\mu}}\right] \tag{13}$$

For MDOF structures, the above equations result in an error of 0·4–5 per cent in the tuning ratio and 0·5–0·8 per cent in the damping ratio. If more accurate parameters are needed, a searching procedure similar to that applied before should be used.

Equation (13) indicates that the best location for a TMD is where it results in the largest ξ, i.e. at the level where Φ and consequently the damping in the TMD and in the first two modes are maximum. Since in most

Table V. Optimum TMD parameters for the three MDOF structures

No. of storeys	Mass ratio μ	Damping ratio (first mode) β	Tuning ratio f	TMD damping ratio ξ	ξ_1	ξ_3	$\frac{\xi+\beta}{2}$	Φ at the top storey
10	0·050	0·02	0·9302	0·3253	0·1759	0·1758	0·1727	1·359
6	0·075	0·05	0·9070	0·4139	0·2437	0·2435	0·2170	1·327
3	0·100	0	0·8701	0·3694	0·1955	0·1953	0·1847	1·231

cases, the first mode dominates the response, it is desirable to locate the TMD at the top floor where the displacement amplitude of the first mode is the largest. Similar observations have also been reported by Villaverde[17].

Numerical studies

To demonstrate the effectiveness of the proposed procedure for computing the optimum TMD parameters, the ten- and six-storey buildings with and without TMDs were analysed using four recent earthquake accelerograms. The records include: the 90° component of Corralitos Eureka Canyon Road accelerogram and the 90° component of Capitola Fire Station accelerogram from the Loma Prieta earthquake, 1989, and the 90° component of Santa Monica City Hall Grounds accelerogram and the 90° component of Arleta Nordhoff Avenue Fire Station accelerogram from the Northridge earthquake, 1994. The displacement and acceleration responses for the structures with and without TMDs are presented in Table VI for the ten-storey building and in Table VII for the six-storey building. It is observed that for the ten-storey building (Table VI), a TMD with an effective mass ratio of 0·05 (a mass ratio of 0·04 when considering the actual mass rather than the generalized mass of the structure) and a damping ratio $\beta = 0{\cdot}02$ results in a considerable reduction in displacements and accelerations (up to 48 per cent). Similar observations are made for the six-storey building with an effective mass ratio of 0·075 (a mass ratio of 0·062 when considering the mass of the structure) and a damping ratio $\beta = 0{\cdot}05$. As expected, the higher the intrinsic damping in the structure, the larger is the mass required to achieve approximately the same level of response reduction.

PRACTICAL CONSIDERATIONS

TMDs are relatively easy to implement in new buildings and in retrofitting existing ones. They do not require an external power source to operate and do not interfere with vertical and horizontal load paths like some other passive devices do. TMDs can also be combined with active control mechanisms to function as a hybrid system, with the TMD serving as back-up in the case of failure of the active device. From the numerical studies presented in previous sections, it is evident that TMDs are effective in reducing earthquake-induced vibrations. TMDs, however, require a considerable mass to achieve a sizeable reduction in the response, especially for structures with large damping ratios. The following practical considerations are noteworthy:

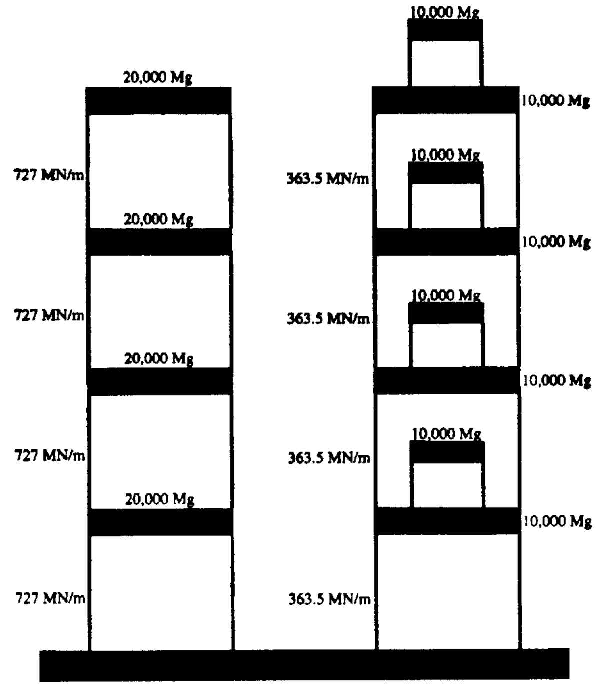
1. For structures with low damping ratios, TMDs with small mass ratios can be effective in reducing the response. The existing mechanical equipment, often placed on the roof, may be used as TMDs by mounting them with springs and dampers with proper stiffness and damping. Another possibility is using blocks of concrete, steel, or lead as used in John Hancock Tower and Citicorp Center Office Building. In any case, TMDs will experience large displacements which must be accounted for in the design.
2. For structures with high damping ratios, TMDs with large mass ratios are required to significantly reduce the response. In such cases, the use of roof equipment or addition of heavy blocks will not provide the mass necessary to introduce sufficient damping in the predominant modes of vibration. The top floor itself, however, can provide the required mass. The concept of 'expendable top storey' introduced by Jagadish *et al.*[16] or the 'energy absorbing storey' presented by Miyama[21] is an effective alternative where the top floor acts as a vibration absorber for the other floors of the building. Although this concept may work effectively, the top floor may experience large displacements. The top floor should, therefore, have sufficient strength and ductility to account for large displacements.

Table VI. Responses of the ten-storey building with and without a TMD with a mass ratio of 0·05

	Corralitos, 1989				Capitola, 1989				Santa Monica, 1994				Arleta, 1994			
	No TMD		With TMD		No TMD		With TMD		No TMD		With TMD		No TMD		With TMD	
Level	x_{max} (m)	a_{max} (g)	x_{max} (m)	a_{max} (g)	x_{max} (m)	a_{max} (g)	x_{max} (m)	a_{max} (g)	x_{max} (m)	a_{max} (g)	x_{max} (m)	a_{max} (g)	x_{max} (m)	a_{max} (g)	x_{max} (m)	a_{max} (g)
Top	0·396	2·43	0·271	1·67	0·257	2·04	0·180	1·26	0·483	2·29	0·450	1·51	0·398	1·64	0·203	0·84
9	0·337	1·99	0·235	1·32	0·214	1·12	0·160	1·03	0·443	1·56	0·408	1·26	0·381	1·04	0·182	0·59
8	0·239	1·34	0·177	0·88	0·204	1·40	0·146	1·03	0·402	1·80	0·362	1·34	0·354	1·19	0·163	0·61
7	0·163	0·64	0·122	0·58	0·199	1·44	0.132	0·74	0·376	1·47	0·315	0·98	0·306	0·86	0·136	0·59
6	0·194	1·02	0·137	0·72	0·182	1.06	0·109	0·71	0·361	1.40	0·263	1·10	0·290	0·88	0.138	0·51
5	0·240	1·61	0·165	1·09	0·160	1·32	0·111	0·73	0·322	1·80	0·218	1·31	0·268	1·20	0·134	0·78
4	0·267	1·85	0·169	1.14	0·150	1·37	0·098	0·93	0·284	1·53	0·170	1·09	0·236	1·14	0·117	0·56
3	0·244	1·80	0·156	1·12	0·136	1·10	0·085	0·88	0·226	1·40	0·145	1·13	0·188	0·88	0·089	0·59
2	0·185	1·49	0·126	1·02	0·122	1·43	0·071	0·84	0·164	1·29	0·100	0·81	0·132	0·97	0·059	0·51
1	0·102	1·04	0·070	0·78	0·072	1·30	0·039	0·79	0·092	1·59	0·051	1·12	0·070	0·90	0·032	0·61

Table VII. Responses of the six-storey building with and without a TMD with a mass ratio of 0·075

	Corralitos, 1989				Capitola, 1989				Santa Monica, 1994				Arleta, 1994			
	No TMD		With TMD		No TMD		With TMD		No TMD		With TMD		No TMD		With TMD	
Level	x_{max} (m)	a_{max} (g)	x_{max} (m)	a_{max} (g)	x_{max} (m)	a_{max} (g)	x_{max} (m)	a_{max} (g)	x_{max} (m)	a_{max} (g)	x_{max} (m)	a_{max} (g)	x_{max} (m)	a_{max} (g)	x_{max} (m)	a_{max} (g)
Top	0·277	2·56	0·186	1·65	0·135	1·70	0·110	0·88	0·111	1·68	0·088	1·57	0·158	1·67	0·121	0·96
5	0·254	2·09	0·167	1·50	0·116	1·24	0·093	1·00	0·096	1·20	0·067	1·06	0·136	1·17	0·104	0·84
4	0·210	1·75	0·135	1·15	0·100	1·72	0·077	1·03	0·081	1·51	0·054	1·07	0·108	1·19	0·087	0·81
3	0·167	1·93	0·103	1·02	0·084	1·61	0·057	0·94	0·063	1·37	0·045	1·00	0·083	1·37	0·066	0·73
2	0·118	1·74	0·068	0·91	0·059	1·45	0·037	0·58	0·042	1·71	0·031	1·31	0·053	1·48	0·043	0·87
1	0·060	1·43	0·033	0·76	0·031	1·39	0·020	0·86	0·022	1·62	0·018	1·19	0·026	1·24	0·021	0·81



Properties of the Fundamental Mode of the Mega-Structure:

$\omega_f = 0.333$ Hz

$M_f = 35{,}737$ Mg

First Mode Shape: 1.241
(normalized to a 1.091
unit participation 0.810
factor) 0.431

Parameters of the Sub-Structures:

Feng and Mita (all levels):

$k = 5{,}480$ KN/m

$c = 6{,}411$ KN.s/m

This Study:

Level	k (KN/m)	c(KN.s/m)
Top	23,665	18,452
3	25,233	16,756
2	28,638	13,249
1	34,457	7,733

Figure 7. Properties of the mega-structure with and without control

VIBRATION ABSORBERS FOR TALL BUILDINGS

Feng and Mita[8] assert that, for tall buildings, the large stiffness of the structural components and the dominance of bending deformations prevent the use of conventional damping devices. The large stiffness requires a significant number of damping devices to achieve the desired damping ratios and the dominant bending deformations render damping devices which utilize shear deformation ineffective. Consequently, they proposed an innovative vibration control system to reduce the dynamic response of tall buildings to wind and seismic loads. Their proposed system takes advantage of the 'mega-substructure configuration' used in the design of tall buildings. The substructures, consisting of several floors, serve to dissipate energy without added masses. The details of such systems are discussed in Reference 8. They arrive at the parameters of the substructures by using a two-degree-of-freedom system and minimizing the mean square response of the main mass to a white noise ground acceleration for seismic analysis and to a white noise force excitation for wind analysis. For seismic loading, they give the following absorber parameters[‡] which ignore the effects of damping and the mode shapes of the structure:

$$f = \frac{\sqrt{1-\mu/2}}{1+\mu} \quad \text{and} \quad \xi = \frac{1}{2}\sqrt{\frac{\mu(1-\mu/4)}{(1+\mu)(1-\mu/2)}} \tag{14}$$

where the mass ratio μ is defined as the ratio of the substructure mass to the floor mass instead of the substructure mass to the generalized mass of the fundamental mode. Feng and Mita used this procedure to compute the response of a 200 m tall building with a damping ratio of 0·02 in each mode to the S00E

‡ Feng and Mita[8] define their damping ratio in terms of the natural frequency of the structure, whereas in this paper the damping ratio is defined in terms of the natural frequency of the damper. With appropriate substitution, it can be shown that the second of equation (14) and the damping expression in Table 3 of Feng and Mita[8] are identical

Table VIII. Response of the mega-structure to the S00E component of El Centro scaled to a peak ground velocity of 0·25 m/s

	No control		Mega subconfiguration Feng and Mita[8]*				With control using proposed procedure			
	Mega-structure		Mega-structure		Sub-structure		Mega-structure		Sub-structure	
Level	x_{max} (m)	a_{max} (g)	x_{max} (m)	a_{max} (g)	Stroke_{max} (m)	a_{max} (g)	x_{max} (m)	a_{max} (g)	Stroke_{max} (m)	a_{max} (g)
Top	0·357	0·28	0·156		0·195	0·02	0·105	0·13	0·074	0·04
3	0·319	0·25					0·102	0·09	0·070	0·03
2	0·215	0·19					0·091	0·12	0·067	0·04
1	0·122	0·25					0·055	0·16	0·062	0·02

* Responses of lower stories not reported

component of El Centro, the Imperial Valley earthquake, 1940, scaled to a peak ground velocity of 0·25 m/s. Figure 7 shows a schematic of the building with and without the mega-substructure configuration along with their properties and the dynamic characteristics of the first mode. The figure shows the modelling of the substructures as SDOF systems attached to the mega-structure. They assumed that the substructures have the same mass as the floors they are attached to, resulting in a mass ratio $\mu = 1$. Their procedure results in a considerable reduction in the response of the building (Table VIII).

To test the effectiveness of the method proposed in this paper for computing the absorber parameters, the building with the same configuration was analysed using equations (12) and (13) to select the parameters. The mass ratio μ was computed by equation (11). Unlike the Feng and Mita procedure[8] where f and ξ are the same at each substructure level, the parameters computed by equations (12) and (13) are different for each level because of the influence of the fundamental mode shape. The lower the substructure in the building, the smaller is the fundamental mode shape amplitude and, consequently from the equations, the larger the stiffness and the smaller the damping. The results of the analysis are presented in Table VIII along with those reported by Feng and Mita. The results show that using the proposed parameters further reduces the mega-structure displacements (from 0·156 to 0·105 m for the top floor), and the substructure displacements (from 0·195 to 0·074 m for the top floor stroke). The proposed parameters, however, do increase the substructure accelerations (from 0·02g to 0·04g for the top floor) as compared with those reported by Feng and Mita.

CONCLUSIONS

The overall objective of this paper was to determine the optimum parameters of tuned mass dampers that result in a considerable reduction in the response to earthquake loading. The criterion used is to find, for a given mass ratio, the tuning and damping ratios of the device that would result in approximately equal damping in the first two modes of vibration. The optimum TMD parameters for SDOF and MDOF structures are presented in tabular and equation forms. It was found that the equal damping ratios in the first two modes are greater than the average of the damping ratios of the lightly damped structure and the heavily damped TMD. Consequently, the fundamental modes of vibration are more heavily damped. The proposed method was used to select the parameters of TMDs for several SDOF and MDOF structures subjected to a number of earthquake excitations. The results indicate that using the proposed TMD parameters reduces the displacement and acceleration responses significantly (up to 50 per cent).

The method was also applied to a vibration control system proposed by Feng and Mita[8] for tall buildings, referred to as mega-substructure configuration, where the substructures in the mega-structure serve as vibration absorbers. Further reductions in the displacement response of the mega-structure and substructures were achieved using the method proposed in this paper. The acceleration response of the substructure, however, was increased compared to that reported by Feng and Mita.

The results also show that in order for TMDs to be effective, large mass ratios must be used, especially for structures with higher damping ratios. The top floor with appropriate stiffness and damping can act as a vibration absorber for the lower floors. The safety and functionality of top floors, however, may present problems since the top floor may experience large displacements.

APPENDIX I

Table IX. Earthquake records used in the statistical study

Earthquake	Mag.	Station name	Epicentral distance (km)	Comp.	Peak accel. (g)
Imperial Valley	6·7	El Centro Valley	11·6	S00E	0·348
05/18/1940		Irrigation District		S90W	0·214
Northwest California	5·8	Ferndale City Hall	56·3	S44W	0·104
10/07/1951				N46W	0·112
Kern County	7·7	Pasadena - Caltech	127·0	SOOE	0·047
06/21/1952		Athenaeum		S90W	0·053
		Taft Lincoln School	41·4	N21E	0·156
		Tunnel		S69E	0·179
		Santa Barbara Court	88·4	N42E	0·089
		House		S48E	0·131
		Holywood Storage	120·4	SOOW	0·055
		Basement		N90E	0·044
Eureka	6·5	Ferndale City Hall	40·0	N44E	0·159
12/21/1954				N46W	0·201
San Francisco	5·3	San Francisco Golden	11·2	N10E	0·083
03/22/1957		Gate Park		S80E	0·105
Hollister	5·7	Hollister City Hall	22·1	S01W	0·065
04/08/1961				N89W	0·179
Borrego Mountain	6·4	El Centro Valley	67·3	SOOW	0·130
04/08/1968		Irrigation District		S90W	0·057
Long Beach	6·3	Vernon CMD Bldg.	50·5	S08W	0·133
03/10/1933				N82W	0·155
Lower California	7·1	El Centro Valley	66·4	SOOW	0·160
12/30/1934		Irrigation District		S90W	0·182
Helena Montana	6·0	Helena, Montana	6·2	SOOW	0·146
10/31/1935		Carrol College		S90W	0·145
1st Northwest California	5·5	Ferndale City Hall	55·2	N45E	0·144
09/11/1938				S45E	0·089
Northern California	5·2	Ferndale City Hall	43·1	N44E	0·054
09/22/1952				S46E	0·076
Wheeler Ridge, California	5·9	Taft Lincoln School	42·8	N21E	0·065
01/12/1954		Tunnel		S69E	0·068
Parkfield, California	5·6	Cholame, Shandon,	56·1	N05W	0·355
06/27/1966		California Array #5		N85E	0·434
		Chalome, Shandon,	53·6	N50E	0·053
		California Array #12		N40W	0·064
		Temblor, California #2	59·6	N65W	0·269
				S25W	0·347

Table IX. (*Contd.*)

Earthquake	Mag.	Station name	Epicentral distance (km)	Comp.	Peak accel. (g)
San Fernando 02/09/1971	6·4	Pacoima Dam	7·3	S16E	1·172
				S74W	1·070
		8244 Orion Blvd.	21·1	N00W	0·255
		Los Angeles, California		S90W	0·134
		250 E First Street	41·4	N36E	0·100
		Basement, Los Angeles		N54W	0·125
		Castaic Old Ridge	29·5	N21E	0·315
		Route		N69W	0·270
		7080 Hollywood Blvd.	33·5	N00E	0·083
		Basement, Los Angeles		N90E	0·100
		Vernon CMD Bldg.	48·0	N83W	0·107
				S07W	0·082
		Caltech Seismological	34·6	S00W	0·089
		Lab., Pasadena		S90W	0·193

APPENDIX II

a_{max}	maximum absolute acceleration
A	system matrix
c	damping coefficient of TMD
f	tuning ratio of TMD
g	acceleration of gravity
i	unit imaginary number
I	identity matrix
k	stiffness coefficient of TMD
m	mass of TMD
M	generalized mass in an MDOF structure
$[M]$	mass matrix
n	number of degrees of freedom
r	counter for eigenvalues and mode shapes
T	natural period
x_{max}	maximum relative displacement

Greek letters

β	damping ratio of structure
ϕ	fundamental modal shape
Φ	modal amplitude at the location of TMD
λ	eigenvalues
μ	mass ratio of TMD
ω_0	natural or fundamental frequency of the structure
ω_t	natural frequency of TMD
ω_1, ω_3	natural frequencies in the first two complex modes
ξ	damping ratio of TMD
ξ_1, ξ_3	damping ratios in the first two complex modes

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